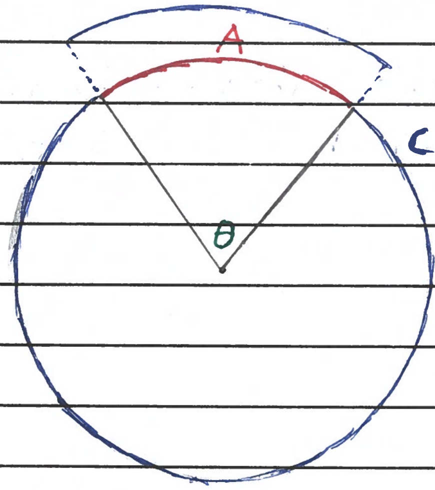
Date: \ \

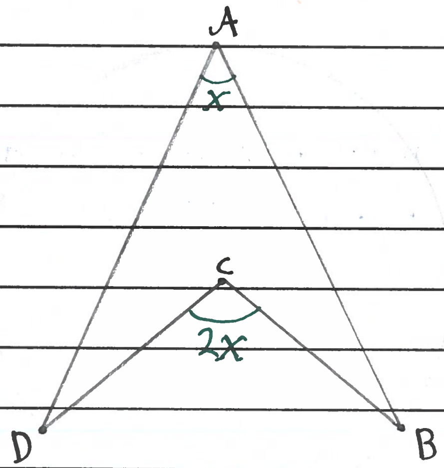
Circle Theorems

$$\frac{\text{Arc length}}{\text{circumference}} = \frac{\text{Angle}}{360°}$$



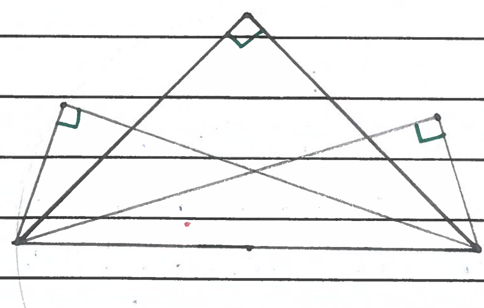
Theorem #1: (Angle at the centre)

The angle at the centre is twice the angle at the circumference.



Theorem #2: (Angles in a semi-circle)

The angle in a semi-circle is a right angle.



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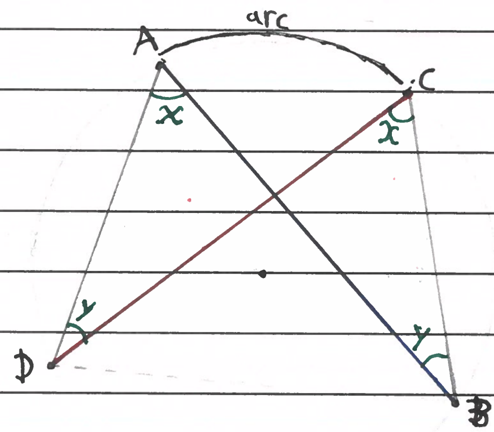
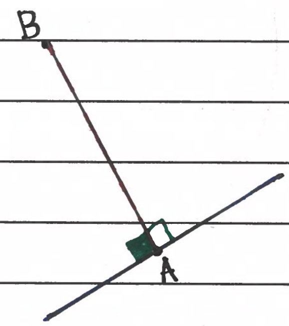
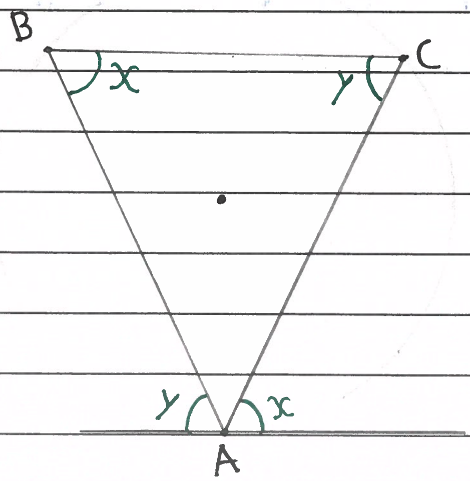
Theorem #3: (Angles in the same segment)
Angles subtended by an arc or chord in the same segment are equal.



Theorem #4: (Tangent of a circle)
The angle between a tangent and radius is 90 degrees.



Theorem #5: (Alternate angle)
The angle that lies between a a tangent and a chord is equal to the angle subtended by the same chord in the alternate segment.



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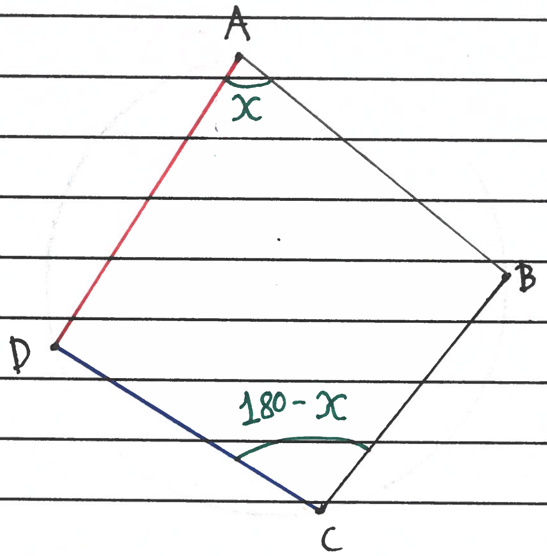
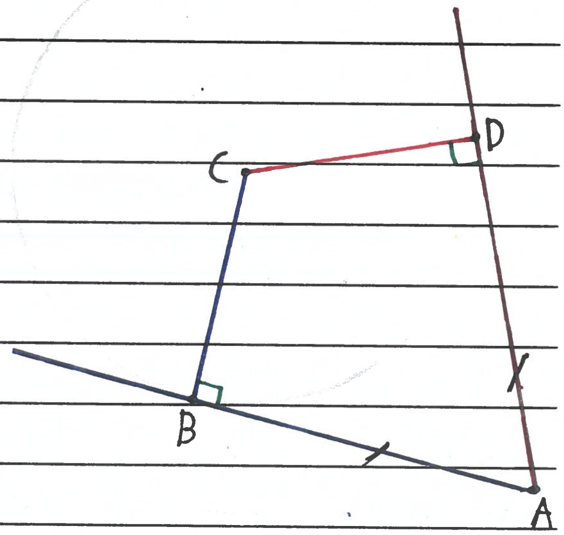
Theorem #6: (Cyclic quadrilateral)

The opposite angles in a cyclic quadrilateral total 180 degrees (supplementary).



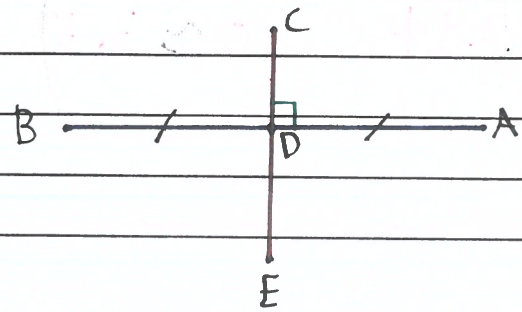
Theorem #7: (Two tangents of circle)

Tangents which meet at the same point are the same length.



Theorem #8: (Chord of a circle)

The perpendicular from the centre of a circle to a chord bisects the chords.



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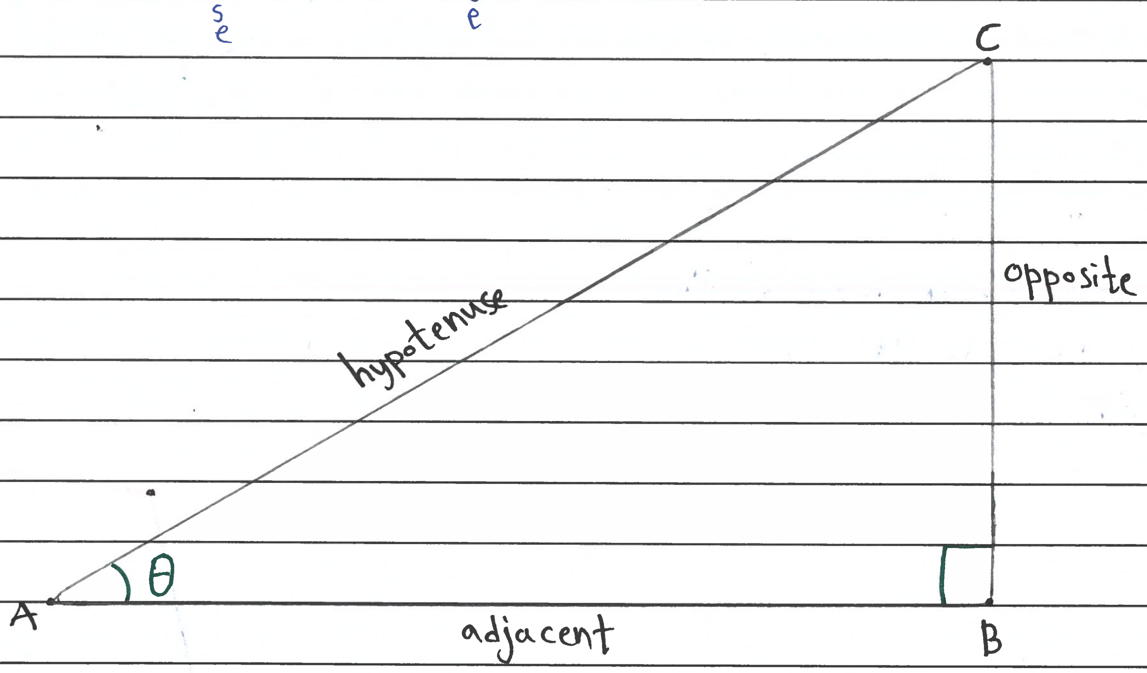
Trigonometry

Sin **O**pposite **H**ypotenuse **C**osin **A**djacent **H**ypotenuse **T**an **O**pposite **A**djacent



1#: Solving for a side		2#: Solving for angle θ (Theta)
Tan will be used as example:		Tan will be used as example:
$\tan 35° = \frac{5}{a}$	$\tan 35° = \frac{o}{5}$	$\theta = \tan^{-1}(\frac{6}{13})$
$a = \frac{5}{\tan 35}$	$5 \cdot \tan 35) = o$	$\theta \approx 24.76$
$a \approx 7.14$	$o \approx 3.5$	

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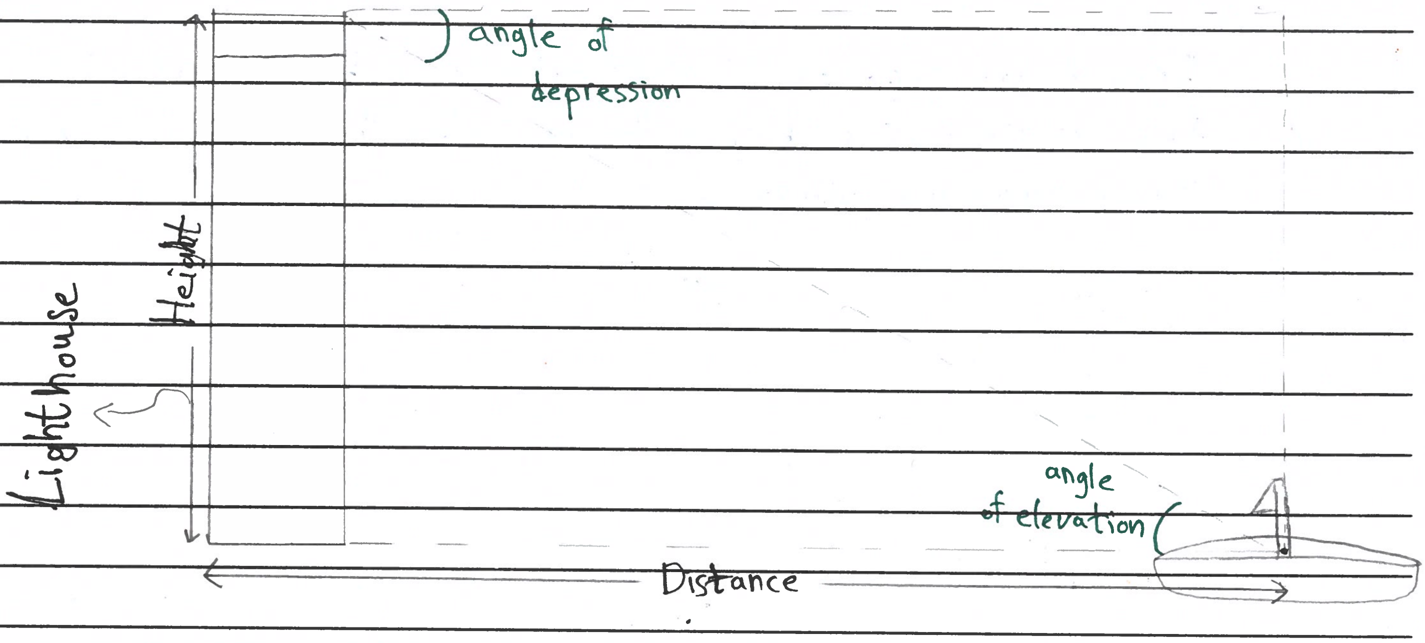
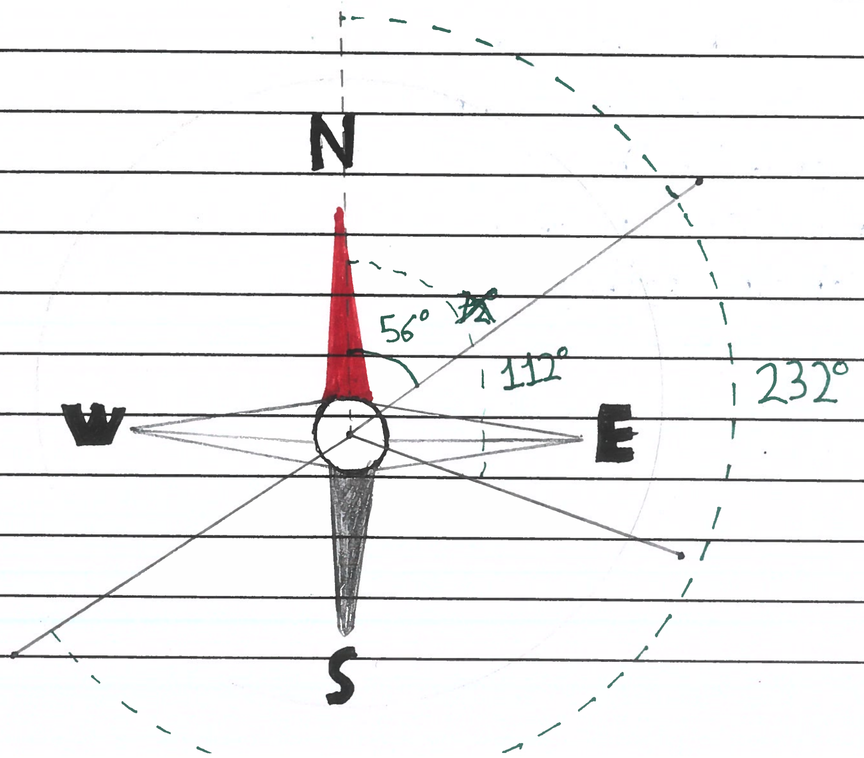
Angles of depression and elevation



True Bearings

Bearing is direction referenced from true North, this angle is allways measured clockwise as shown in this figure. The angle is allways written in 3 digits "x x x"



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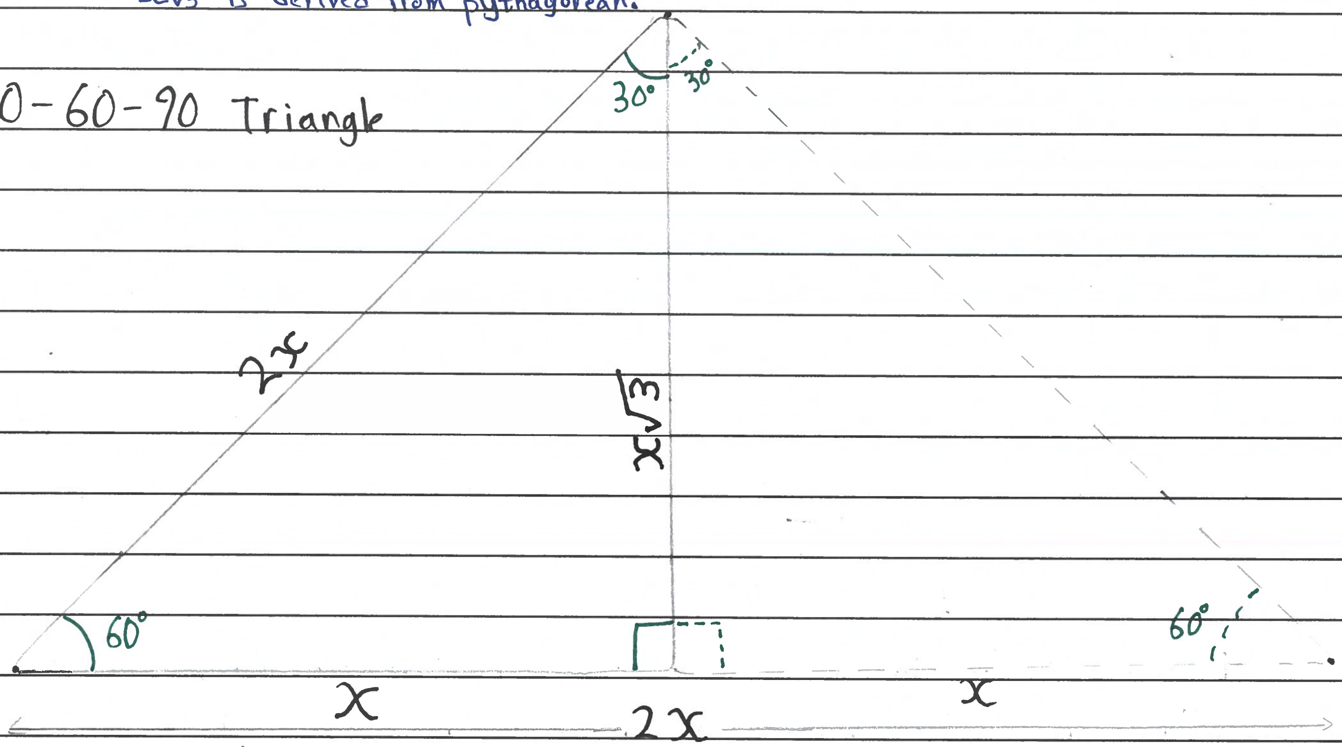
Special Right Triangles

Since if you mirror a 30-60-90 triangle it becomes an equilatural triangle, since x is half of the base side, the other side is $2x$. $x\sqrt{3}$ is derived from pythagorean.

30-60-90 Triangle



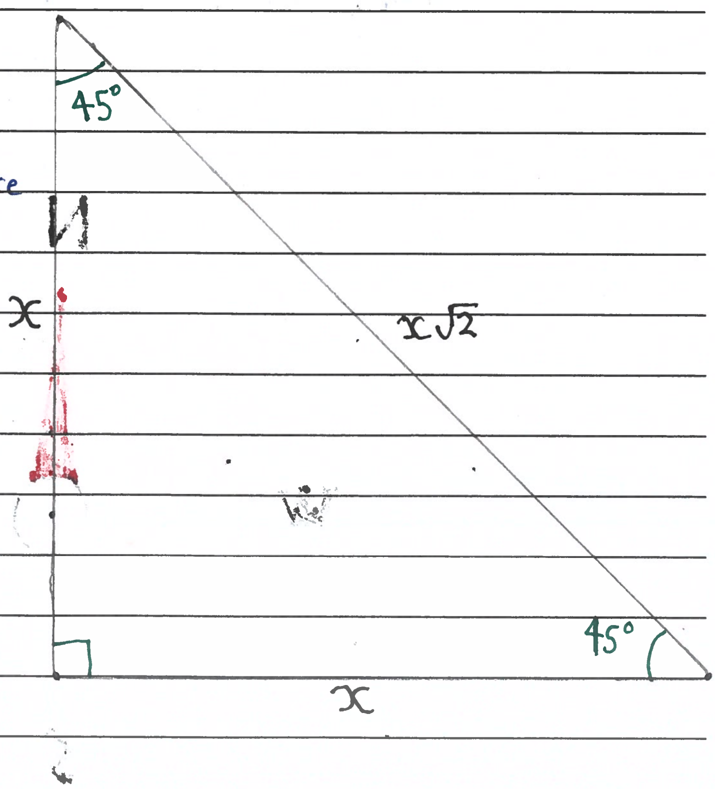
45-45-90 Triangle

Since this triangle has two equal angles it is an isosceles triangle, hence two sides are equal. In this case, the third side is $x\sqrt{2}$ as it was calculated with the pythogrean theorem:

$x^2+x^2=h^2$

$2x^2=h^2$

$x\sqrt{2}=h$



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Indices

Rules of indices:

1. $a^m \times a^n = a^{m+n}$	2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{m \cdot n}$	4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	6. $a^0 = 1$
7. $a^{-n} = \frac{1}{a}$	8. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Scientific notation

1. $a \times 10^n, \quad 1 \leq |a| \leq 10$ | 2. $(a \times 10^m) \times (b \times 10^n) = ab \times 10^{m+n}$ | $\frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m+n}$

4. $a \times 10^m + b \times 10^n = a \times 10^m + b \times 10^{m+(n-m)} = (a + b \times 10^{n-m}) \times 10^m$

Radicals

1. $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ | 2. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ | 3. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

4. $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$ | 5. $\frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$

6. $\frac{1}{\sqrt{a}+b} \times \frac{\sqrt{a}-b}{\sqrt{a}-b} = \frac{\sqrt{a}-b}{(\sqrt{a})^2 - b^2}$ | 7. $\frac{1}{\sqrt{a}-b} \times \frac{\sqrt{a}+b}{\sqrt{a}+b} = \frac{\sqrt{a}+b}{(\sqrt{a})^2 - b^2}$

8. $\sqrt{a} + \sqrt{a} = 2\sqrt{a}$ 9. $\sqrt{a} - \sqrt{a} = 0$ 10. $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$

11. $\sqrt{a} + \sqrt{b} = \sqrt{a} + \sqrt{b}$ 12. $\frac{\sqrt{a}}{\sqrt{a}} = 1$ 13. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

Date: \ \

Coordinate Geometry

Useful equation:

Distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Midpoint = $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

Linear Function Forms

Point-slope form:

$y-y_1=m(x-x_1)$

Intercept Form

$\frac{y}{b}+\frac{x}{a}=1$

General Form

$Ax+by+c=0$

$y=\frac{\text{Rise}}{\text{Run}}x+b$

Parallel Linear Function

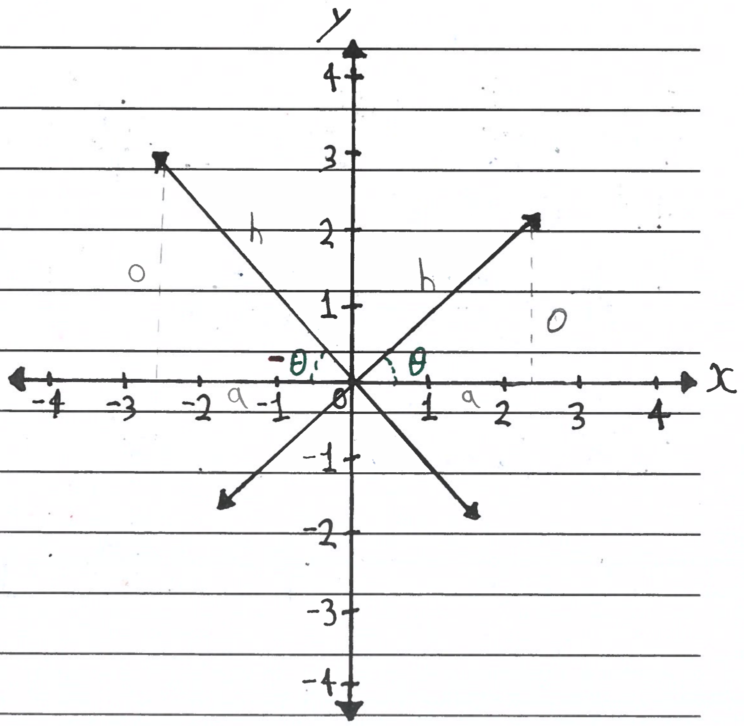
$y=\frac{\text{Rise}}{\text{Run}}x+b$ ("b" must be different)

Perpendicular linear Function

$y=\frac{-\text{Run}}{\text{Rise}}x+b$ (negative reciprocal)

Relation of tan to slope:

The angle between a positive slope line and a horizontal line (or x-axis) is equal to $\tan\theta$. This is because $\tan\theta$ is opposite (rise) over adjacent (run) or $\tan\theta=\frac{\text{Rise}}{\text{Run}}$. This also means that a negative slope is equal to $\tan -\theta$. Keep in mind that this property along with distance equation, midpoint equation, parallel, and perpendicular may come in a single question.



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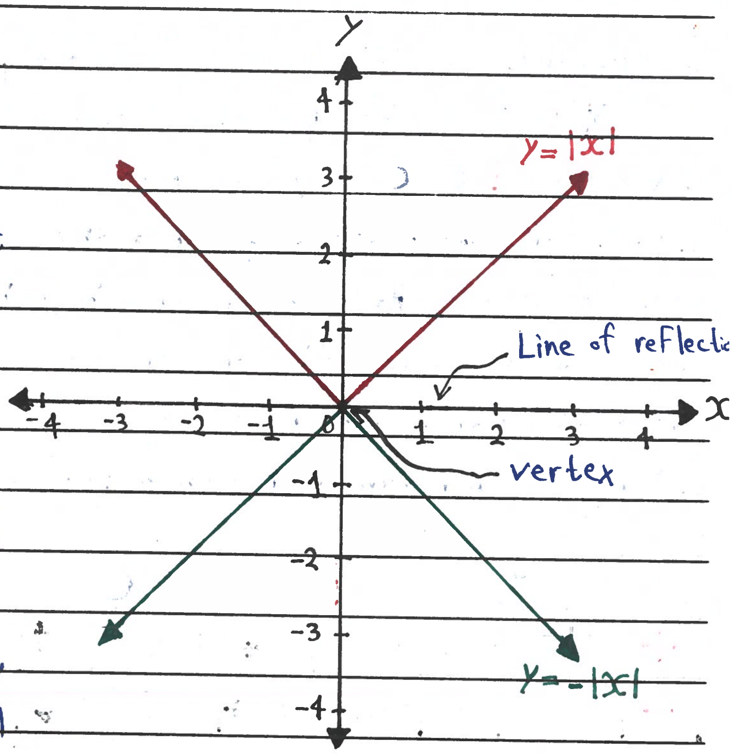
Absolute value

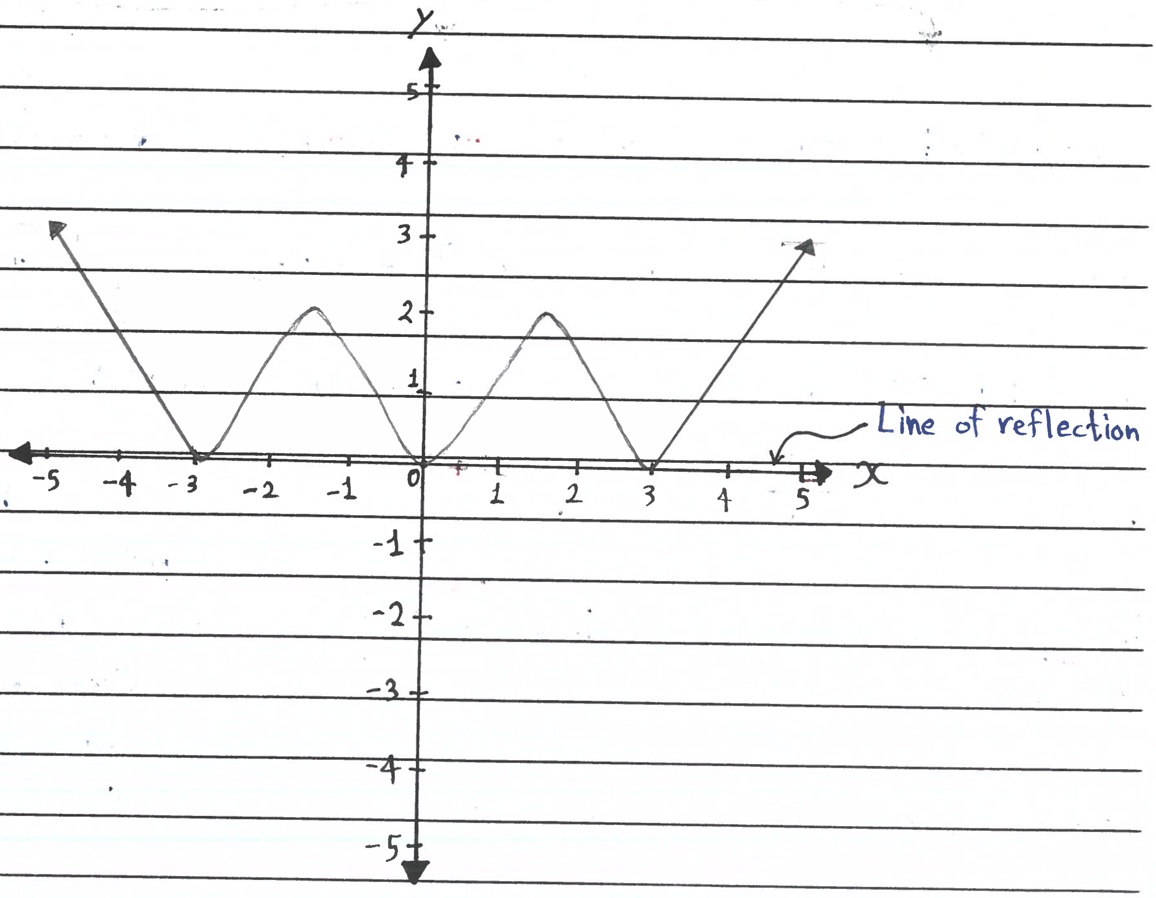
Absolute value rules:

1. $y=|x|$ is the parent function
2. The range is positive only
3. Unless $y=-|x|$ then only negative output
4. The graph is allway inverted at the line of reflection (or x value of vertex)



Graphing:

1. Draw as a normal line ($y=|x| \rightarrow y=x$)
2. Solve for x-intercept ($y=0$) to get x-coordinate of vertex or line of symmetry
3. Mirror one of the points from the original line around line of symmetry.



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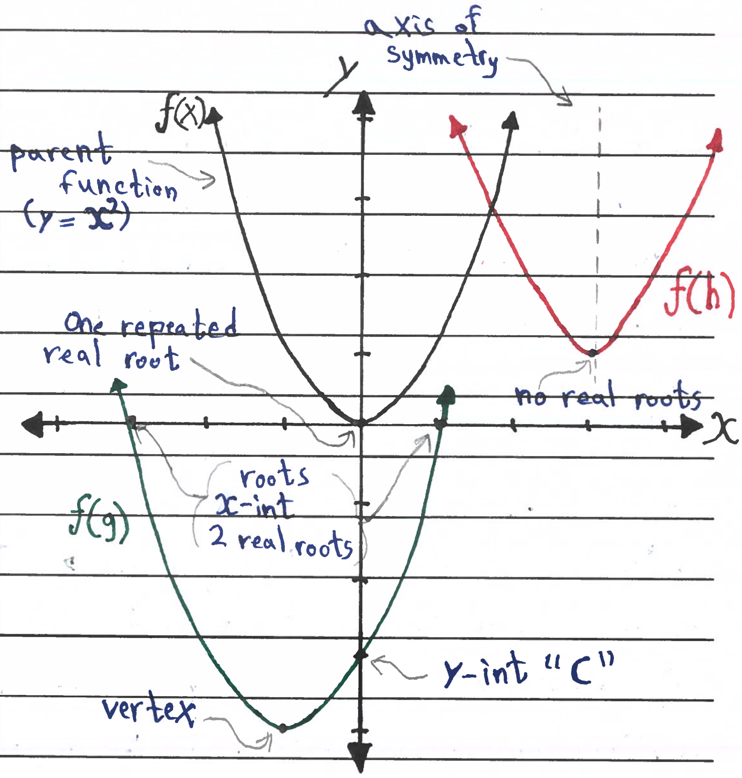
Quadratic functions

1. Quadratic functions are seconed degree
2. $f(x) = x^2$ is the parent function
3. $y = ax^2 + bx + c$ is general form
4. Roots are the points of parabola that intersect x-axis. Has to be one from the graph
5. Axis of symmetry is the line around which the parabola is reflected
6. Vertex is minimum or maximum point

	a	b	c
+	Concave up	vertex shift ←	Graph shift ↑
−	Concave down	vertex shift →	Graph shift ↓



Quadratic Function Forms and Graphing

Standard form

$y = ax^2 + bx + c$

1. y-intercept: $f(0) =$ "c"
2. Roots: $f(x) = 0$ (factoring)
3. Axis of symmetry: $\frac{-b}{2a}$
4. vertex y-coordinate: $f\left(\frac{-b}{2a}\right)$
5. Mirror point

Roots form

$y = a(x-p)(x-q)$

1. $f(0)$ or $a \cdot p \cdot q$
2. $x = p$, $x = q$
3. $\frac{p+q}{2}$
4. $f\left(\frac{p+q}{2}\right)$
5. Mirror point

vertex form

$y = a(x-h)^2 + K$

1. $f(0)$
2. $y = 0$ (x subject) $x = h \pm \sqrt{\frac{-K}{a}}$
3. (h, K)

(Note that "h" in the vertex form is reversed from the graph, so allways reverse the sign when graphing

5. Mirror point

Date: \ \

Expansion and factorization

Expansion:

Perfect square trinomials:	Binomial expansion:	Difference of two squares:
$(x+b)^2 = x^2 + 2bx + b^2$	$(x+b)(x+d) = x + (b+d)x + bd$	$(x+b)(x-b) = x^2 - b^2$
$(x-b)^2 = x^2 - 2bx + b^2$		

Factorization:

Greatest common factor:	Grouping:	Two numbers: $(a=1)$
$6x^2 + 12x = 0$	$x^2 + 5x + 6 = 0$	$x^2 + 7x + 12 = 0$
$6x(x+2) = 0$	$x^2 + 2x + 3x + 6 = 0 \rightarrow$ GFC	$\boxed{3}$ and $\boxed{4}$ + × $(x+3)(x+4)$
	$(x+2)(x+3) = 0$	

Cross multiplication: $(a \neq 1)$:

$$\boxed{2x^2 + 7x + 3 = 0}$$

$$(2x+1)(x+3)$$

$(2x \; + \; 1)$ $+x$

$(x \; + \; 3)$ $6x$

The signs must satisfy the original equation

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If all factorization methods faill, use quadratic formulla as a last resort.

$b^2 - 4ac$ is called discriminant (Δ)

If the discriminant is negative, there are no real roots.

Explaination	Expression
Original quadratic equation	$9x^2 + 30x - 7 = 0$
Greatest common factor "9"	$9(x^2 + \frac{10}{3}x) - 7 = 0$
Divide coefficient of x by two to get perfect square. Square it	$\frac{5}{3} \xrightarrow{\div 2}$ $(\frac{5}{3})^2 \rightarrow \frac{25}{9}$
Get the negative of square of second term and remove it from Bracket	$9(x+\frac{5}{3})^2 + 9(\frac{-25}{9}) - 7 = 0$
Simplify $9(\frac{-25}{9})$ and subraction	$9(x+\frac{5}{3})^2 - 25 - 7 = 0$
Simplify subtraction	$9(x+\frac{5}{3})^2 - 32 = 0$
Add 32 to both sides	$9(x+\frac{5}{3})^2 = 32$
Divide both sides by 9	$(x+\frac{5}{3})^2 = \frac{32}{9}$
Squar root both sides	$x + \frac{5}{3} = \pm\sqrt{\frac{32}{9}}$
Isolate variable x	$x = -\frac{5}{3} \pm \sqrt{\frac{32}{9}}$

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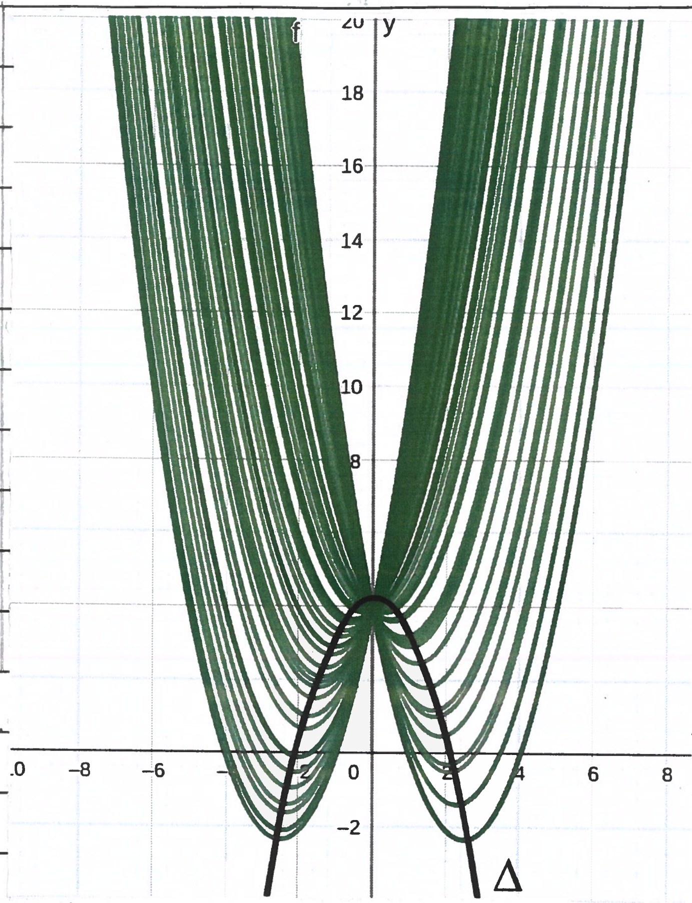
Discriminant

Discriminant:

The discriminant ($b^2 - 4ac$ or Δ) determines a "sliding" effect on the original function. This sliding exhibits a parabolic shape as seen in the feagure. The green parabolas are the possible possitions of the function at its vertex. The black parabola is what you get when you graph the function's discriminant. The discriminant also determines the number of real roots.



Discriminant	Roots
$\Delta > 0$	Two real roots
$\Delta = 0$	One real root
$\Delta < 0$	No real roots

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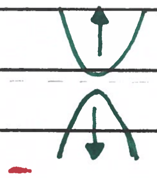
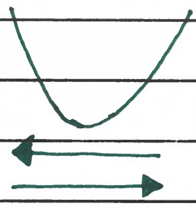
Quadratic transformations



Reflection on x-axis

Reflection on y-axis



$$y = -af(-b(x-h)) \pm K$$

→ Horizontal Transformation:

- b → horizontal dialation
  - Stretch: $0<a<1$
  - Compression: $a>0$
- h → horizontal translation:
  - Left: $\boxed{+h}$, shifts function left
  - Right: $\boxed{-h}$, Shifts function right

→ Vertical Transformation

- a → vertical dialation:
  - Stretch: $a>0$
  - Compression: $0<a<1$
- K → vertical translation:
  - Up: $\boxed{+K}$, Shifts function up
  - Down: $\boxed{-K}$, shifts function down